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ELECTROMAGNETIC OSCILLATING TWO STREAM INSTABILITY

OF PLASMA WAVES

by

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ELECTROMAGNETIC OSCILLATING TWO STREAM INSTABILITY

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ABSTRACT

Near the resonance ($\omega_o \approx \omega_p$) a high amplitude plasma wave decays into a purely growing electromagnetic perturbation and two Langmuir wave sidebands. The instability has a growth rate $\gamma \approx \omega_p k_o^2 |v_o|^2 / 8 k^2 v_e^2$ for a Langmuir pump of finite wavenumber $k_o > k v_e/c$, and $\gamma \approx \omega_p |v_o|^2 / 16 c^2$ for a laser light pump. In the latter case, the instability has an inhomogeneity threshold $|v_o|^2 \omega_p L/2 c^3 = 1$ and is a source of spontaneous generation of magnetic field.

I. INTRODUCTION

Oscillating two stream instability, in which a plasma wave decays into a zero frequency electrostatic mode and two Langmuir wave sidebands has been known for some time and shown to be an important nonlinear process leading to soliton formation, plasma collapse, etc. In this paper we have investigated the electromagnetic oscillating two stream instability of electrostatic and electromagnetic plasma waves of finite wave number by which a pump wave decays into a purely growing electromagnetic mode and two sidebands, viz., Langmuir waves of shorter wavelengths. For an electrostatic pump, i.e., a Langmuir wave $(\vec{k}_o || \vec{E}_1)$, the low frequency perturbation has a finite longitudinal component of electric vector (i.e., $\vec{k} \cdot \vec{E} \neq 0$) whereas for an electromagnetic pump $(\vec{k}_o \perp \vec{E}_o)$ or a dipole pump, the perturbation is purely electromagnetic (i.e., $\vec{k} \cdot \vec{E} = 0$). In the latter case, it is similar to the Weibel instability in an anisotropic plasma and leads to spontaneous generation of magnetic field. Bodner and Eddleman[1] had some time ago pointed out this instability of intense laser radiation by considering a dipole pump and a homogeneous plasma. Here we have studied the electromagnetic oscillating two stream instability of Langmuir waves of finite wavenumber; $k_o \neq 0$ introduces major qualitative and quantitative effects on the parametric instability. In the case of an electromagnetic pump, i.e., a laser radiation, we have studied this instability including the effect of plasma inhomogeneity and shown its importance as a source of self-generated magnetic fields in laser-fusion experiments.

Self-generated magnetic fields of a few megagauss have been observed in a number of experiments and computer simulations of laser produced plasmas.[2-6] The origin of these fields is believed to be due to: (1) resonance absorption,[5-8] in which the density oscillations due to the p-polarized light couple with

the drift velocity to produce a time average current and hence, the magnetic field; (2) ponderomotive force,[9-12] which contains a curl free term due to collisions; (3) mutually perpendicular gradients in density and temperature $\underline{\nabla}n \times \underline{\nabla}T$; (4) thermal instability[13] and (5) Weibel instability of counter streaming electron beams. For high power density lasers the electromagnetic oscillating two stream instability should become an important source of magnetic field generation.

The mechanism of the instability is as follows. A high amplitude plasma wave $(\omega_0, \vec{k}_0)$ propagating through a plasma produces an oscillating drift velocity of electrons $\vec{v}_0$ in the direction of the electric field $\vec{E}_0$. We perturb the system by an electromagnetic perturbation $(\omega, \vec{k})$; $(\vec{k} \perp \vec{E}_0)$ and the magnetic vector $\vec{B}$ is perpendicular to both $\vec{k}$ and $\vec{E}_0$. For an electromagnetic pump $\vec{k}_0 || \vec{k}$, where as $\vec{k}_0 || \vec{E}_0$ for an electrostatic pump. In the former case the magnetic field of the perturbation couples with the oscillatory drift velocity of electrons to produce a $\vec{v}_0 \times \vec{B}$ force driving the high frequency sidebands, viz., electrostatic plasma waves. The sidebands couple with the pump to produce a low frequency current driving the instability. For an electrostatic pump, the low frequency perturbation has finite density fluctuations also, which lead to an important nonlinearity producing the sidebands. The sidebands interact with the pump to produce a low frequency ponderomotive force as well as a transverse current driving the instability. In both the cases the sidebands are coupled through the relativistic velocity dependence of mass and hence the coupling coefficient for the parametric decay is modified.

In Section II we have studied the electromagnetic oscillating two stream instability of Langmuir waves of finite wavenumber. In Section III, the instability analysis is extended to an electromagnetic pump, viz., laser radiation, including the effect of a density gradient. A discussion of results is given in Section IV.

II. INSTABILITY OF LANGMUIR WAVES

We consider the propagation of a Langmuir wave $\hat{x}\, E_o\, \exp{-i(\omega_o t - k_o x)}$ in an uniform plasma. This produces an oscillatory velocity $\vec{v}_o = e\, \vec{E}_o/m\, i\, \omega_o$, a density oscillation $n_o = n_o^o\, e\, k_o\, E_o/m\, i\, \omega_o^2$, and decays into a low frequency electromagnetic perturbation $\vec{E}(\vec{k},\omega)$, $\vec{B} = c\, \vec{k} \times \vec{E}/\omega$ and two high frequency electrostatic sidebands $\phi_j(\omega_j,\vec{k}_j)$; $j = 1,2$, $\omega_{1,2} = \omega \mp \omega_o$, $\vec{k}_{1,2} = \vec{k} \mp \vec{k}_o$, $\vec{k}\,||\,\hat{z}$ and $k \gg k_o$. The pump and sidebands produce high frequency current densities, $J_{-2x}(\omega - 2\omega_o) = -n_1\, e\, v_o^*/2$ and $J_{+2x}(\omega + 2\omega_o) = -n_2\, e\, v_o/2$ also, which produce a magnetic field $B_{\mp 2y} = 4\pi\, i\, k\, J_{\mp 2x}/c(k^2 - 4\omega_o^2/c^2)$, important for deriving the sidebands.

The response at $(\omega,\vec{k})$ and $(\omega_{1,2},\vec{k}_{1,2})$ is obtained by solving the equations of continuity and motion,

$$\frac{\partial n}{\partial t} + \underline{\nabla} \cdot (n\vec{v}) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(m\vec{v}) + \vec{v} \cdot \underline{\nabla}(m\vec{v}) = -e\vec{E} - \frac{e}{c}\vec{v} \times \vec{B} - \frac{T}{n}\underline{\nabla}n , \tag{2}$$

$$m = m_o(1 + \vec{v} \cdot \vec{v}/2c^2).$$

We expand the quantities $\vec{v}$ and n with subscripts o, 1, 2 referring to the pump and two sidebands and the quantities without a subscript referring to $(\omega,\vec{k})$ component. For ω_1, $\vec{k}_1$ component Eq. (1) gives

$$n_1 \simeq -k_o v_o^* n/2\omega_1 + n_o^o\, k\, v_{1z}/\omega_1 . \tag{3}$$

v_{1z} is obtained from Eq. (2) retaining the relativistic effects through the first term on the left-hand-side and $\vec{v} \times \vec{B}$ nonlinearity through the

second term on the right. The last term in Eq. (2) can be expressed in terms of ϕ_1 by using the Poisson's equation $k^2\phi_1 = -4\pi en_1$. Solving these coupled equations along with the similar equations for the upper side band $\omega_2, \vec{k}_2$ we obtain

$$\epsilon_1\phi_1 = -\frac{|v_o|^2}{8c^2}\phi_2 + \frac{2\pi}{k^2c^2}v_o J_x + \frac{k_o v_o}{\omega_o}\frac{2\pi ne}{k^2} \tag{4}$$

$$\epsilon_2\phi_2 = -\frac{|v_o|^2}{8c^2}\phi_1 - \frac{2\pi v_o}{k^2c^2}J_x - \frac{k_o v_o}{\omega_o}\frac{2\pi ne}{k^2} \tag{5}$$

where

$$\epsilon_j = 1 - \frac{\omega_p^2}{\omega_j^2} - \frac{k^2 v_e^2}{\omega_o^2} + \frac{|v_o|^2}{8c^2}\frac{k^2 + 4\,\omega_o^2/c^2}{k^2 - 4\,\omega_o^2/c^2} \quad,$$

$$j = 1,2, \quad \omega_p^2 = 4\pi n_o^o e^2/m_o(1 + 3|v_o|^2/8c^2).$$

In writing these equations we have expressed $\vec{B} = \hat{y}\,4\pi i\,J_x/kc$. In Eqs. (4) and (5) the first terms on RHS are due to relativistic mass correction, second terms due to $\vec{v}_o \times \vec{B}$ force and the last terms due to $n\vec{v}_o$ nonlinearity in the equation of continuity. The last term vanishes for a dipole pump ($k_o = 0$).

For $\omega, \vec{k}$ component Eqs. (1) and (2) give

$$v_x = \frac{eE_x}{m_o i\omega} , \tag{6}$$

$$v_z = \frac{kv_e^2}{\omega}\frac{n}{n_o^o} + \frac{eE_z}{m_o i\omega} + \frac{1}{2\omega}(-k_o v_o v_{1z} + k_o v_o^* v_{2z}), \tag{7}$$

$$n \simeq -\frac{eE_z n_o^o}{m_o i k v_e^2} - \frac{k_o v_o}{2v_e^2}\frac{e(\phi_1 - \phi_2)n_o^o}{m\omega_o} . \tag{8}$$

Here we have ignored the relativistic corrections as $\omega, \vec{k}$ is not a normal mode. Using the Poisson's equation and Eq. (8), one obtains,

$$n = \frac{n_o^o e}{T} \frac{k_o v_o}{2\omega_o} (\phi_1 - \phi_2) \frac{\omega_{pi}^2}{\omega^2 \epsilon} \tag{9}$$

where

$$\epsilon \simeq \frac{\omega_p^2}{k^2 v_e^2} - \frac{\omega_{pi}^2}{\omega^2} \tag{10}$$

The low frequency current density may be written as

$$J_x = -n_o^o e v_x - n_1 e v_o/2 - n_2 e v_o^*/2 = - \frac{n_o^o e^2 E_x}{mi\omega} + \frac{k^2 v_o}{8\pi}(\phi_1 - \phi_2). \tag{11}$$

Using Eq. (11) in the wave equation,

$$\nabla^2 E_x + \frac{\omega^2}{c^2} E_x = - \frac{4\pi i\omega}{c^2} J_x$$

one obtains,

$$E_x = \frac{i\omega}{2c^2} \frac{v_o(\phi_1 - \phi_2)}{1 + \omega_p^2/k^2 c^2} \quad , \tag{12}$$

$$J_x = \frac{v_o(\phi_1 - \phi_2)k^2}{8\pi(1 + \omega_p^2/k^2 c^2)} \quad . \tag{13}$$

Employing Eq. (13) in Eqs. (4) and (5), we obtain the nonlinear dispersion relation

$$\epsilon_1' \epsilon_2' = |v_o|^4 \psi/16c^4 \tag{14}$$

$$\psi = [\frac{1}{1 + \omega_p^2/k^2 c^2} - \frac{1}{2} + \frac{k_o^2 c^2}{k^2 v_e^2} \frac{\omega_{pi}^2}{\omega^2 \epsilon}]^2 \quad ,$$

$$\epsilon_{1,2}' = 1 - \frac{\omega_p^2}{\omega_{1,2}^2} + \Delta$$

$$\Delta = - \frac{k^2 v_e^2}{\omega_o^2} + \frac{|v_o|^2}{4c^2} [\frac{1}{1 + \omega_p^2/k^2 c^2} + \frac{k^2 + 4\omega_o^2/c^2}{2(k^2 - 4\omega_o^2/c^2)} + \frac{k_o^2 c^2}{k^2 v_e^2} \frac{\omega_{pi}^2}{\omega^2 \epsilon}]^2$$

Defining $\delta = \omega_p - \omega_o(1 + \Delta)^{1/2}$, Eq. (14) takes the form

$$\omega^2 = \delta^2 - \frac{\omega_o^2}{4} \frac{|v_o|^4}{16c^4} \psi \; . \tag{15}$$

The largest growth occurs for $\delta = 0$ i.e., for $\Delta \simeq 0$ because $\omega_o \overset{\sim}{\sim} \omega_p$ or

$$\frac{k^2 v_e^2}{\omega_o^2} \simeq \frac{|v_o|^2}{4c^2} [\frac{1}{1 + \omega_p^2/k^2 c^2} + \frac{k^2 + 4\omega_o^2/c^2}{2(k^2 - 4\omega_o^2/c^2)} + \frac{k_o^2 c^2}{k^2 v_e^2} \frac{\omega_{pi}^2}{\omega^2 \epsilon}] \tag{16}$$

$$\overset{\sim}{\sim} \frac{2\omega}{\omega_o}$$

Eq. (15) can be solved in different limits. For $k_o^2 c^2/k^2 v_e^2 < 1$, the last terms in the expressions of ψ and Δ can be neglected and the growth rate $\gamma = -i\omega$ turns out to be

$$\gamma \simeq \omega_p |v_o|^2/16c^2 \; , \tag{17}$$

the same expression as obtained by Bodner and Eddleman[1] in the limit of a dipole pump. The low frequency mode is purely electromagnetic in the limit of $k_o \to 0$.

In the opposite limit of $k_o^2 c^2/k^2 v_e^2 > 1$, i.e., a pump wave of substantial wavenumber, the last terms in ψ and Δ are predominant; these terms appear through the density fluctuations associated with the low frequency mode, viz., the last terms in Eqs. (4) and (5). On further assuming $\omega < 2\omega_p m/m_i$ (m_i is the mass of ions) the dispersion relation yields,

$$\gamma \simeq \frac{\omega_o}{8} \frac{|v_o|^2 k_o^2}{v_e^2 k^2} \; . \tag{18}$$

It must be mentioned here that the low frequency mode is a mixed mode with magnetic field $B_y \simeq 2 \omega_{pi}^2 E_z/\omega(\omega\omega_p)^{1/2} > E_z > E_x$. For $\omega > 2 \omega_p m/m_i$, the dispersion relation becomes a bicubic equation,

$$\omega^6 - \frac{k^4 c_s^4}{\omega_{pi}^4} \omega^4 + \frac{2k^2 k_o^2 c_s^4}{\omega_{pi}^2} \frac{|v_o|^2}{v_e^2} \omega^2 - \frac{3}{64} \frac{|v_o|^4}{v_e^4} k_o^4 c_s^4 = 0 \ . \tag{19}$$

The roots are complex and have a finite real frequency. Eq. (19) may be solved numerically.

III. INSTABILITY OF LASER RADIATION:

A SOURCE OF SELF-GENERATED MAGNETIC FIELDS

Here we consider the colinear propagation of laser light pump $\hat{x} E_o \exp-i(\omega_o t - k_o z)$ and all the decay waves along the z axis, i.e., $\vec{k}_o || \vec{k} || \hat{z}$. The z components of the low frequency ponderomotive force and the current density and hence, the density oscillations vanish, i.e., the low frequency mode is purely electromagnetic. The nonlinearity driving the low frequency perturbation comes from $\vec{J} = -n_1 e\vec{v}_o/2 - n_2 e\vec{v}_o^*/2$ [the same as Eq. (11)] and the sidebands are driven by the $\vec{v}_o \times \vec{B}$ force. Sideband potentials are given by Eqs. (4) and (5) with n = 0. The nonlinear dispersion relation, for k >> k_o, turns out to be the same as Eq. (14) with ψ and Δ having $k_o \equiv 0$. The uniform medium growth rate is then given by Eq. (17), viz., $\gamma = \omega_p |v_o|^2/16c^2$. For a Nd:glass laser $\gamma \simeq 3 \times 10^{11} \sec^{-1}$ at $4 \times 10^{15}(1 - \omega_p^2/\omega_o^2)^{1/2}$ W/cm^2, i.e., the growth time is $\sim 3p$ sec. It may be worthwhile mentioning here that the growth rate of electrostatic oscillating two stream instability is given by $\gamma = \mathrm{Im}\omega$, $\omega^2 - k^2 c_s^2 - \omega^4/(\omega^2 - \omega_{pi}^2) = 2\omega_{pi}^2 \beta_o \delta/(\omega^2 - \delta^2)$ where $\beta_o = k^2 |v_o|^2/4\omega_p$. At large power densities, the growth rate maximizes for $\delta \simeq \gamma_{max}$ and $\gamma_{max} \simeq k^2 \lambda_D^2 \omega_p |v_o|^2/8v_e^2$.

Now we consider the effect of a density gradient on this instability. All the waves are taken to propagate along the density gradient i.e., z axis. Using Eqs. (4) and (5) and writing $J_x = \frac{kc}{4\pi i} B_y$, $E_1 = -\frac{\partial}{\partial z} \phi_1$, $E_2 = -\frac{\partial}{\partial z} \phi_2$ one obtains

$$\frac{d^2}{dz^2} E_1 + \left(\frac{\omega_1^2 - \omega_p^2}{v_e^2} - \frac{|v_o|^2}{v_e^2} \frac{\omega_o^2}{8c^2}\right) E_1 + \frac{|v_o|^2}{v_e^2} \frac{\omega_o^2}{8c^2} E_2 = -\frac{v_o}{2c} \frac{\omega_o^2}{v_e^2} B_y \qquad (20)$$

$$\frac{d^2 E_2}{dz^2} + \left(\frac{\omega_2^2 - \omega_p^2}{v_e^2} - \frac{|v_o|^2}{v_e^2} \frac{\omega_o^2}{8c^2}\right) E_2 + \frac{|v_o|^2}{v_e^2} \frac{\omega_o^2}{8c^2} E_1 = \frac{v_o}{2c} \frac{\omega_o^2}{v_e^2} B_y \ . \qquad (21)$$

Eq. (11), in compliance with the Faraday's and Ampere's laws, gives

$$\frac{d^2 B_y}{dz^2} - \frac{\omega_p^2}{c^2} B_y = - \frac{v_o}{2c} \frac{d^2}{dz^2} (E_1 - E_2).$$

(22)

Eqs. (20) - (22) are very similar to those obtained for oscillating two stream instability suggesting that the electromagnetic instability is absolute.[15] To calculate the threshold for the instability, we take $\omega = 0$ (the growth vanishes at the threshold) and assume $\omega_p^2 = (\omega_o^2 - \frac{|v_o|^2}{4c^2} \omega_o^2)(1 + Z/L)$. Then Eqs. (20) - (22) take the form

$$\frac{d^2 S}{dz^2} - (\frac{Z}{L\lambda_D^2} - \frac{|v_o|^2}{4c^2 \lambda_D^2}) S = 0$$

(23)

$$\frac{d^2}{dz^2} D - \frac{Z}{L\lambda_D^2} D = - \frac{v_o}{c\lambda_D^2} B_y$$

(24)

$$\frac{d^2}{dz^2} B_y - \frac{\omega_p^2}{c^2} B_y = - \frac{v_o}{2c} \frac{d^2}{dz^2} D$$

(25)

where $\lambda_D^2 = v_e^2/\omega_p^2$, $S = E_1 + E_2$, $D = E_1 - E_2$. Eqs. (23) - (25) can be solved employing the Fourier transform technique (cf. Liu[15])

$$E_1 = \frac{1}{4\pi} \int_{-\infty}^{\infty} dk [\exp-i(kz + Lk^3\lambda_D^3/3)]$$

$$\cdot \left\{ a_2 \exp(i L \frac{|v_o|^2}{4c^2} k) + a_1 \right.$$

$$\left. \exp[i L \frac{|v_o|^2}{2c^2} (k - \frac{\omega_p}{c} \tan^{-1} \frac{kc}{\omega_p})] \right\}$$

(26)

$$E_2 = \frac{1}{4\pi} \int_{-\infty}^{\infty} dk [\exp-i(kz + Lk^3\lambda_D^3/3)]$$

$$\cdot \left\{ a_2 \exp(i L \frac{|v_o|^2}{4c^2} k) - a_1 \right.$$

$$\exp\left[i \left[\frac{|v_o|^2}{2c^2} \left(k - \frac{\omega_p}{c} \tan^{-1} \frac{kc}{\omega_p} \right] \right] \right. \quad . \tag{27}$$

At large values of $-z(-z > L|v_o|^2/2c^2)$ the integrals can be evaluated by the saddle point method with saddle points at $k = \pm(-z/L)^{1/2}\lambda_D^{-1}$, corresponding to ingoing and outgoing waves. Since the source of these waves is situated at $z \simeq 0$, there cannot be any incoming wave at $z \to -\infty$. This demands[15]

$$|v_o|^2 \omega_p L/2c^3 = (2n + 1); \; n \text{ is an integer.}$$

The corresponding threshold power density of a Nd:glass laser is $\sim 5 \times 10^{14}$ W/cm^2 for a density scale length of 100 μm. Here we have taken into account the enhancement of the pump field (which goes as an Airy function) in the vicinity of resonance, $E_o \sim (L/\lambda_{em})^{1/4} \cdot E_o$ (vacuum).

As the amplitude of the instability grows with time, the magnetic field B attains large values and modifies the dispersion relation for plasma waves, increasing the frequency shift δ,

$$\delta = (\omega_p^2 + \omega_c^2)^{1/2} - \omega_o(1 + \Lambda)^{1/2}$$

where $\omega_c = eB/mc$. The instability would be completely stabilized when $\delta \sim \omega_p |v_o|^2/16c^2$, i.e., $\omega_c \sim \omega_p |v_o|/2\sqrt{2}c$. This corresponds to $B \sim 2M$ Gauss at laser intensity $\sim 10^{15}$ W/cm^2. However, there may be other saturating mechanisms also for this instability.
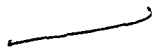
IV. DISCUSSION

Electromagnetic oscillating two stream instability constitutes an efficient channel of decay of Langmuir waves to shorter wavelengths and may play an important role in the saturation of beam-plasma instability. The decay waves propagate almost perpendicular to the pump wave and hence, one should consider the effects of finite beam size on this instability. The effect of finite k_o, wave number of the pump, is to excite density fluctuations along with purely growing electromagnetic perturbation. These fluctuations provide strong interaction between the sidebands and the pump and thereby enhance the growth rate γ; $\gamma \sim \omega_p k_o^2 |v_o|^2 / 8 v_e^2 k^2$ for $k_o > k \, v_e/c$.

For a high power laser radiation, this instability is an important source of spontaneous generation of magnetic fields, which have been observed in a number of experiments. The effect of finite k_o is negligible and the growth rate $\gamma \simeq \omega_p |v_o|^2 / 8 c^2$. For a Nd:glass laser, the growth turns out to be a few pico second at $\sim 10^{15}$ W/cm^2. The density threshold for this instability is $\sim 5 \times 10^{14}$ W/cm^2 for a density scale length ~ 100 μm. As the amplitude of the instability (i.e., of the magnetic field) grows, the four wave resonance is detuned and the instability is stabilized at a level $\omega_c \sim \omega_p v_o/2\sqrt{2}c$. This corresponds to the magnetic fields of a few megagauss at laser intensities $\sim 10^{15}$ W/cm^2.

ACKNOWLEDGMENTS

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